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Analysis of Gain and Stability in Josephson Traveling-Wave Parametric Amplifiers with Non-Sinusoidal Current–Phase Relations

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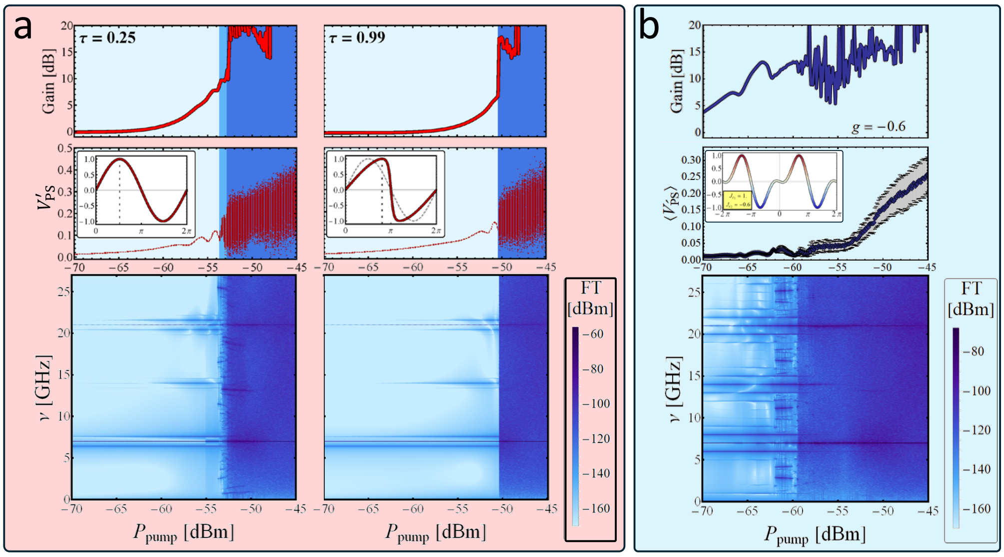
Abstract: Numerical simulations show that non-sinusoidal current–phase relations strongly shape JTWPA gain, stability, and chaos thresholds. Second-harmonic engineering and resonant phase matching enhance amplification, but also increase frequency mixing and spurious tones, requiring careful design optimization.

Josephson traveling-wave parametric amplifiers (JTWPAs) are key enabling components for broadband, low-noise microwave amplification in quantum technologies, where gain, spectral purity, and dynamical stability must be simultaneously optimized. Here we present a unified numerical study of JTWPA operation beyond the standard sinusoidal-junction picture, focusing on the impact of non-sinusoidal current–phase relations under realistic driving conditions [1-3]. By varying pump power, signal frequency, and dc bias current, we map the transition from regular amplification to complex and chaotic dynamics through gain profiles, phase-space trajectories, Poincaré sections, and Fourier spectra. We show that CPR engineering provides a powerful route to tailor device performance: transparency-induced skewness modifies both the maximum attainable gain and the width of the stable operating region [1], while a second-harmonic contribution enables a nontrivial optimization of the amplification process, with gains up to about 13 dB even without dispersion engineering [2]. We further demonstrate that resonant phase matching can enhance the gain also in the non-sinusoidal case, although this improvement is accompanied by stronger mixing processes and the appearance of additional spurious tones [3]. Overall, these results establish a consistent picture of how anharmonic Josephson nonlinearities govern gain, stability, and spectral cleanliness, and provide practical guidelines for the design of next-generation JTWPAs for quantum readout and precision measurements.



a. **Transparency-dependent CPR**: Gain (top panel), Poincaré sections (middle panel), and Fourier spectra of the output voltage (bottom panel) versus the pump power level, P_{pump}, at different junction transparencies, i.e, τ = {0.25 and 0.99}.
b. **Second-harmonic CPR**: Gain (top panel), Poincaré sections (middle panel), and Fourier spectra of the output voltage (bottom panel) versus the pump power level, P_{pump}, for a CPR $I(\varphi) = J_{c_1} \sin\varphi + J_{c_2} \sin 2\varphi$ at $(J_{c_1}, J_{c_2}) = (1, -0.6)$.

References

[1] C. Guarcello, et al., *Driving a Josephson Traveling Wave Parametric Amplifier into chaos: Effects of a non-sinusoidal current–phase relation*, Chaos, Solitons and Fractals **189**, 115598 (2024).
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[3] C. Guarcello, et al., *Properties of Josephson Traveling Wave Parametric Amplifier With a Non-Sinusoidal Current-Phase Relation and Resonant Phase Matching*, IEEE Trans. Appl. Supercond. **36**, 1700405 (2026).